

JTMS-MAT-13: Numerical Methods**Assignment Sheet 3. Released: April 04, 2024****Due: April 16, 2024****Exercise 1 [3+3+3+3 Points]:**

Let $f(x) = 3x^3 - 4x^2 + 4x - 1$, and consider the starting points $x_0 = 0$, and $x_1 = 1$.

- Check whether bisection, secant, and Newton's method can be applied.
- Apply three steps of the bisection, Newton's and secant method. (For Newton's method start from $x_0 = 0$)
- Find the roots analytically and compare the errors of the results you computed in b).
- Which of the above method's are expected to converge and why?

Exercise 2 [8 Points]:

Starting with $(0, 0)$ apply two iterations of Newton's method to solve the system of non-linear equations

$$\begin{aligned} -x^2 + x + 4y &= -2 \\ (x - 1)^2 + (2y - 3)^2 &= 5 \end{aligned}$$

Exercise 3 [5+5 Points]:

Consider the tabulated data

x	-7	-6	0	5
y	-23	-12	-6	1

- Derive the polynomial $p_\ell(x)$ in Lagrange form that interpolates the values y at the nodes x .
- Use the polynomial and compute the interpolated value at $x = -1$. Apply Aitken's algorithm and recompute the interpolated value.