Constructor University **Spring Semester 2024**

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JTMS-MAT-13: Numerical Methods

Assignment Sheet 2. Released: March 06, 2024
Due: March 18, 2024

Exercise 1 [4+4+2 Points]:

a) Consider:

$$1 - \sqrt{1-\epsilon}.$$

Identify the problematic subtraction in this expression and give an algebraically equivalent formulation of $1 - \sqrt{1-\epsilon}$ that does not suffer from this problematic subtraction for $|\epsilon| \ll 1$. Briefly explain your choice. Hint: Multiply by a fraction (equal to one) which removes the problematic subtraction

b) Use $\epsilon = 0.0002$ and write it in floating point representation with a mantissa of $k = 4$ digits precision and base $b = 10$. Show that your equivalent formulation is much more accurate for $\epsilon = 0.0002$ compared to the original formulation in a).

c) Use the theorem that was shown in class to predict the number of lost significant bits (i.e. in base 2) when executing the problematic subtraction for $\epsilon = 0.0002$.

Exercise 2 [5 Points]:

Find the solution to the system of equations using Gaussian elimination:

$$\begin{aligned} 3x + y - z &= 24 \\ x - y + z &= 2 \\ 2x + y + z &= 0 \end{aligned}$$

Exercise 3 [2+8 Points]:

Let $A = \begin{pmatrix} 2 & 0 & -1 & 2 \\ 2 & 7 & -11 & -2 \\ 4 & 1 & -6 & 2 \\ 2 & 1 & -3 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 6 \\ 5 \\ 9 \\ 3 \end{pmatrix}$.

a) Check if Gaussian elimination with scaled partial pivoting can be applied to solve $Ax = b$.

b) Solve $Ax = b$ for x by Gaussian elimination with scaled partial pivoting.

Exercise 4 [3+4+3 Points]:

Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 3 \\ 2 & 3 & 9 \end{pmatrix}$.

a) Show that matrix A is positive definite.

b) Compute the LU decomposition of A.

c) Compute the Cholesky decomposition of A.