Constructor University Autumn Semester 2024

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Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 1. Released: September 18, 2024
Due: September 28, 2024

Exercise 1 [5+5 Points]:

1. **[5 points]** If $\boldsymbol{v}+\boldsymbol{w}=\begin{bmatrix}3\\1\end{bmatrix}$ and $\boldsymbol{v}-\boldsymbol{w}=\begin{bmatrix}-1\\4\end{bmatrix}$, compute and draw $\boldsymbol{v}$ and $\boldsymbol{w}$.

2. **[5+5+5+5 points]** Describe geometrically all linear combinations of the following collections of vectors. In each case your answer should be one of the following: a line, a plane, or all of $\mathbb{R}^3$.

   (a) $\begin{bmatrix}1\\0\\0\end{bmatrix}$ and $\begin{bmatrix}1\\2\\4\end{bmatrix}$;

   (b) $\begin{bmatrix}2\\4\\1\end{bmatrix}$ and $\begin{bmatrix}6\\12\\3\end{bmatrix}$;

   (c) $\begin{bmatrix}1\\2\\4\end{bmatrix}$ and $\begin{bmatrix}3\\5\\6\end{bmatrix}$ and $\begin{bmatrix}0\\1\\6\end{bmatrix}$;

   (d) $\begin{bmatrix}3\\0\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\2\end{bmatrix}$ and $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

3. **[5+5 points]** The vectors $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ are in an m-dimensional space $\mathbb{R}^m$, and a linear combination $c_1\boldsymbol{a_1}+\cdots+c_n\boldsymbol{a_n}$ is the zero vector.

   (a) Write that statement at the matrix level, that is use the matrix A with the $\boldsymbol{a}$ vectors in its columns and use the column vector $c=(c_1,\ldots,c_n)$.

   (b) Write that statement at the scalar level, i.e. use subscripts and summation notation to add up numbers. The column vector $\boldsymbol{a_j}$ has components $a_{1j}, a_{2j}, \ldots, a_{mj}$.

4. **[5+5 points]**

   (a) What is the sum of the twelve vectors that go from the center of a clock to the hours 1:00, 2:00, ..., 12:00?

   (b) Assume that the distance from the center of the clock to the hours is equal to 1. What is the sum of the twelve vectors that start at 6:00 (at the bottom) and go to the hours 1:00, 2:00, ..., 12:00?

5. **[5 points]** Solve, by any method, the system of linear equations:

$$\begin{aligned} 4x+12y-16z &= 1\\ 12x+40y+-38z &= 2\\ -16x-38y+90z &= 3. \end{aligned}$$