Constructor University **Autumn Semester 2024**

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Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 2. Released: October 2, 2024
Due: October 12, 2024

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1. **[5 points]** A bat and a ball cost 1.10€ in total. The bat costs 1.00€ more than the ball. By solving the system of linear equations, find how much both cost.

2. **[5+5 points]** True or false (give a reason if true or a counterexample if false):
   (a) If $\boldsymbol{u}$ is perpendicular (in three dimensions) to $\boldsymbol{v}$ and $\boldsymbol{w}$, then $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel
   (b) If $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ and $\boldsymbol{w}$, then $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v} + 2\boldsymbol{w}$

3. **[5 points]** Find two non-zero vectors that are perpendicular to $(1, 0, 1)^T$ and to each other.

4. **[5+5 points]** If $\|\boldsymbol{v}\| = 5$ and $\|\boldsymbol{w}\| = 3$, what are the smallest and largest possible values of the following expressions?
   (a) $\|\boldsymbol{v} - \boldsymbol{w}\|$
   (b) $\boldsymbol{v} \cdot \boldsymbol{w}$

5. **[5+5 points]** Suppose A is a 3×3 matrix with ones for every entry.
   (a) Find two independent vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ that solve $A\boldsymbol{x} = \boldsymbol{0}$ and $A\boldsymbol{y} = \boldsymbol{0}$. Write that first equation $A\boldsymbol{x} = \boldsymbol{0}$ (with numbers) as a combination of the columns of A.
   (b) Why is there no third vector, $\boldsymbol{z}$ with $A\boldsymbol{z} = \boldsymbol{0}$, which is independent of $\boldsymbol{x}$ and $\boldsymbol{y}$?

6. **[5+5 points]** A 2×2 matrix of the form

$$R_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad \text{where } \alpha \in \mathbb{R},$$

   is called a *rotation matrix*.
   (a) Show that multiplication by this matrix rotates vectors counter-clockwise by angle α.
   (b) Furthermore, show that $R_{-\alpha}R_\alpha = R_\alpha R_{-\alpha} = I$

   Hint: Express a vector $\boldsymbol{x} \in \mathbb{R}^2$ in polar coordinates, that is $\boldsymbol{x} = r\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$. Then compute $A\boldsymbol{x}$ and use appropriate trigonometric identities.