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## Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 3. Released: October 21, 2024

Due: October 31, 2024

1. [5 points] The rank of a matrix  $A$  is the number of its pivot columns or equivalently, the dimension of  $\text{Col } A$ . Show that the matrix

$$A = \begin{pmatrix} 1 & 4 & -3 \\ 2 & 8 & 6 \\ -3 & -12 & -9 \end{pmatrix}$$

has rank 1.

2. [10 points] Fill out these matrices so that they have rank 1:

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & & \\ 4 & & \end{pmatrix}, \quad B = \begin{pmatrix} & 9 & \\ 1 & & \\ 2 & 6 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} a & b \\ c & \end{pmatrix}.$$

3. [5 points] Let  $A$  be a square matrix. Do  $A^2$  and  $A$  always have the same nullspace?
4. [5+5+5 points] Find a basis of  $\text{Nul } A$  and a basis of  $\text{Col } A$  for the following matrices:

(a)  $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 3 & 5 \end{pmatrix}$

(b)  $\begin{pmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{pmatrix}$

(c)  $\begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix}$

5. [5 points] An  $n \times n$  matrix is invertible, if and only if it has rank  $n$ . Explain, why this is true.
6. [5+5 points] For a given collection of vectors, determine whether they form a basis or not.

(a)  $\left\{ \begin{pmatrix} 3 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 8 \\ -2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} -6 \\ 4 \\ -3 \\ 3 \end{pmatrix} \right\}$

(b)  $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -6 \\ -8 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$