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CTMS-MAT-13: Numerical Methods

Assignment Sheet 1. Released: 12 February 2025
Due: 23 February 2025

Exercise 1 [5+5 Points]:

Let $f\left(x\right) = \sin(\omega x)$ with some positive, real number ω.

a) Show the Taylor series for $f(x)$ around $c = 0$ is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{\left(-1\right)^{n} \left(\omega x\right)^{2n+1}}{(2n+1)!}$$

b) Use the Taylor series truncated after the n-th term to compute approximations of $f\left(\frac{\pi}{2}\right)$ for $n = 1, \ldots, 4$ given $\omega = 1$.

Exercise 2 [5+5 Points]:

a) Show that the Taylor series, with remainder, for $\ln\left(x\right)$ about $x = 1$ can be written as:

$$\ln\left(x\right) = \sum_{k=1}^{n} \frac{\left(-1\right)^{k+1}}{k} (x-1)^{k} + \frac{\left(-1\right)^{n}}{n+1} \frac{(x-1)^{n+1}}{\xi_x^{n+1}}.$$

b) When $\xi_x \in (1, x)$, consider the behaviour of the remainder term in the limit of $n \to \infty$, and derive a bound on x such that the remainder term vanishes in the limit of $n \to \infty$.

Exercise 3 [5+5+3 Points]:

a) Compute the Taylor series for $f(x) = e^{\cos(x)}$ around $c = 0$.
Hint: compute for e^{y} then substitute for y.

b) The Taylor series for $f(x) = \dfrac{1}{1+2x}$ around $c = 0$ *represents* the function for $|x| < \dfrac{1}{2}$. What is the Taylor expansion for $n = 1$ and what is the remainder term? Calculate the number of correct digits for $x = 0.0001$ and $x = -0.0001$.

c) Convert the following from one base to another and write down you calculations as an expansion:

i) $(140)_{10}$ to $(\ldots)_2$

ii) $(10.75)_{10}$ to $(\ldots)_2$

iii) $(111.01001)_2$ to $(\ldots)_8$
Hint: consider $(111)_2$ + $(010)_2$ + $(010)_2$ to get a three digit representation in base 10, then convert each digit from base 10 to base 8.

Exercise 4 [0.5+0.5+0.5+0.5 Points]: Webcolors can be expressed with six base-16 (hexadecimal) digits (two each for the red, green and blue components, in that order) prefixed with `#`. The hexadecimal format uses sixteen distinct symbols, most often the symbols `0-9` to represent values 0 to 9, and `A-F` (or alternatively `a-f`) to represent values from ten to fifteen.

a) How many separate shades are there in each channel of an RGB triplet and in total?

b) How are black and white written in this format?

c) Convert the hexadecimal colour `#00b0ff` into an RGB triplet.

d) CMYK colours encode four channels (cyan, magenta, yellow and black), taking values between 0-100 (inclusive). Are there more possible representations in the CMYK scheme than hexadecimal?