

CTMS-MAT-13: Numerical Methods

Assignment Sheet 2. Released: 26 February 2025

Due: 9 March 2025

Exercise 1 [3+3+1 Points]:

Consider numerical solutions to the quadratic equation

$$ax^2 + bx + c.$$

- With $a = 1$, $b = 200$, $c = 1/1000$, compute the solutions using only four-digit rounding arithmetic for all arithmetic operations.
- State for which solution there will be a loss of significant bits and explain why.
- Compute the absolute errors between the computed solutions and the actual solutions.

Exercise 2 [7 Points]:

Find the solution to the system of equations using Gaussian elimination:

$$-5x + 5y + 10z = -10$$

$$3x - y - 2z = -2$$

$$3x - 6y - 6z = -24$$

Exercise 3 [4+6 Points]:

Let

$$A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 5 & 8 & 6 & 3 \\ 4 & 2 & 5 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}.$$

- Check if Gaussian elimination can be applied to solve $A\mathbf{x} = \mathbf{b}$.
- Solve $A\mathbf{x} = \mathbf{b}$ for \mathbf{x} by Gaussian elimination with scaled partial pivoting.

Exercise 4 [3+4+4 Points]:

Let

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

- Show that matrix A is positive definite.
- Compute the LU decomposition of A .
- Compute the Cholesky decomposition of A .