Constructor University

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CTMS-MAT-13: Numerical Methods

Problem Sheet 1. Released: 19 February 2026

Exercise 1: Let $f(x) = \sin(\omega x)$ with some positive, real number ω.

a) Write down the Taylor polynomials $p_n(x)$ of degree n for $f(x)$ around $c = 0$ for each of the following cases $n = 1, 2, 3, 4$.

b) How large should n be so that $|\sin(x) - p_n(x)| < \varepsilon$ when $\varepsilon = 10^{-4}$ everywhere in the interval $[0, 1]$?

Exercise 2:

a) Show that up to quadratic terms, the Taylor series for $\sqrt{1+x}$ about $c = 0$ can be written as:

$$p_2(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2.$$

b) The remainder term is given by

$$\frac{1}{3!}(x-c)^3 f'''(\xi_x),$$

where ξ_x is between x and $c = 0$. Consider the maximum value of the remainder for all $x \in [0, 1]$, and, by simplifying, show that the upper bound to the remainder term is $1/16$.

c) Considering $x \in \left[0, \frac{1}{2}\right]$, what is the maximum value of the remainder term?

Exercise 3: Compute the Taylor series for $f(x) = e^{\cos(x)}$ around $c = 0$.

Exercise 4: Convert the following from one base to another and write down your calculations as an expansion:

a) $(140)_{10}$ to $(\ldots)_2$

b) $(10.75)_{10}$ to $(\ldots)_2$

c) $(111.01001)_2$ to $(\ldots)_8$
Hint: consider $(111)_2$ + $(010)_2$ + $(010)_2$ to get a three digit representation in base 10, then convert each digit from base 10 to base 8.

Exercise 5: Webcolors can be expressed with six base-16 (hexadecimal) digits (two each for the red, green and blue components, in that order) prefixed with `#`. The hexadecimal format uses sixteen distinct symbols, most often the symbols `0-9` to represent values 0 to 9, and `A-F` (or alternatively `a-f`) to represent values from ten to fifteen.

a) How many separate shades are there in each channel of an RGB triplet and in total?

b) How are black and white written in this format?

c) Convert the hexadecimal colours `#008ce3`, `#00204d` and `#db4f3d` into RGB triplets.

d) CMYK colours encode four channels (cyan, magenta, yellow and black), taking values between 0-100 (inclusive). Are there more possible representations in the CMYK scheme than hexadecimal?

Exercise 6: The *ternary numerical system* uses three as its base. The number $100,000$ in base 10 requires six digits. Its "radix economy" is therefore $10 \times 6 = 60$. For $10,000$ the radix economy is 50.

a) Show that, in base 2, the same number requires 14 digits, so its radix economy is $2 \times 14 =$ 28.

b) Show that, in base 3, it requires 9 digits, so its radix economy is $3 \times 9 =$ 27.

c) Show that, for $100,000$ in base 2, the same number requires 17 digits, so its radix economy is $2 \times 17 =$ 34.

d) Again, for $100,000$, show that, in base 3, the radix economy is 33.