CTMS-MAT-13: Numerical Methods

Exercise 1: Find the solution to the system of equations using Gaussian elimination:

$$-5x + 5y + 10z = 45$$
$$3x - y - z = -3$$
$$3x - 6y + 6z = 15$$

Exercise 2: Let

$$A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 5 & 8 & 6 & 3 \\ 4 & 2 & 5 & 3 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}.$$

a) Check if Gaussian elimination can be applied to solve $A\boldsymbol{x} = \boldsymbol{b}$.

b) Solve $A\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$ by Gaussian elimination with scaled partial pivoting.

Exercise 3: Let

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 5 & 1 \\ 2 & 1 & 6 \end{pmatrix}.$$

a) Show that matrix A is positive definite.

b) Compute the LU decomposition of A.

c) Compute the Cholesky decomposition of A.

Exercise 4:

a) Using

$$\sum_{k=1}^{n-1} k(k+1) = \sum_{k=1}^{n-1} k^2 + \sum_{k=1}^{n-1} k$$
$$= \frac{1}{6}(n-1)n(2n-1) + \frac{1}{2}n(n-1),$$

or otherwise, explain why, for a $(n \times n)$-matrix, Gaussian elimination has $\mathcal{O}\left(n^3\right)$ complexity.

b) Explain why, for a $(n \times n)$-tridiagonal matrix, Gaussian elimination only has $\mathcal{O}(n)$ complexity.