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CTMS-MAT-13: Numerical Methods

Problem Sheet 3. Released: 12 March 2025

Exercise 1: For $f(x) = x^3 - 7x^2 + 14x - 8$, note that $x^* = 1$ is a root.

a) Given this, find the intervals for the other two roots for which the Bisection method will work.

b) With $x_0 = 0.9$ and $x_1 = 1.2$, apply three steps of the bisection, Newton's and secant method. (For Newton's method use $x_0 = 0.9$ as the initial guess)

c) Compare the errors of the results you computed in (**b**).

Exercise 2: For

$$f(x) = x^4 - 8x^2 - 1$$

a) Draw the function and sketch Newton's method

b) With an guess of $x_0 = 1.5$ show that the second iterate is $x_2 \approx -0.2788$.

Exercise 3: Starting with $(x_0, y_0) = (0, 0)$ apply two iterations of the Newton method for the system of non-linear equations

$$\begin{aligned} -x^2 + (x-1)(y+1) + 4y^3 &= 5 \\ (x-2)^2 + (3y-2)^2 &= 5 \end{aligned}$$

to find (x_2, y_2).

Exercise 4: Consider the linear system $A\boldsymbol{x} = \boldsymbol{b}$, with

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 2 & 2 & -3 \\ 0 & 4 & 4 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}.$$

a) Find the inverse to the matrix using Gaussian elimination.

b) Show that after two iterations of the Jacobi scheme, with an initial guess $\boldsymbol{x}_0 = (1, 1, 1)^T$ the error

$$\|\boldsymbol{x}^* - \boldsymbol{x}_2\|_2 = \|(65/12, 139/24, 5/12)\|_2 = \frac{\sqrt{36321}}{24} \approx 7.94086.$$