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CTMS-MAT-13: Numerical Methods

Quiz 1. 25 March 2025

Answer *four* questions only. All questions carry equal marks. Please write you name at the top and please clearly indicate which questions are to be marked.

All trigonometric values are in radians.

Exercise 1: Show that the Taylor series, with remainder, for $\ln(1+x)$ about $x=0$ can be written as

$$\ln(1+x) = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} x^k + \frac{(-1)^n}{n+1} \frac{x^{n+1}}{(1+\xi_x)^{n+1}}.$$

Exercise 2: Express the number 20.3125

a) in binary, i.e. base 2

b) in base 4

c) in base 16 (using hexadecimal format: 0,1,2,3,4,5,6,7,8,9,a,b,c,d,e,f)

Exercise 3: Using Gaussian elimination, or otherwise, what is the solution to the linear system given by $A\boldsymbol{x} = \boldsymbol{b}$ with

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 2 & 2 & -3 \\ 0 & 4 & 4 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}?$$

- ○ $\left(\frac{1}{6}, \frac{25}{12}, \frac{1}{6}\right)^T$
- ○ $\left(\frac{1}{6}, 1, \frac{1}{6}\right)^T$
- ○ $(1,1,1)^T$
- ○ $\left(1, \frac{25}{12}, 1\right)^T$
- ○ $\left(\frac{25}{12}, 1, \frac{25}{12}\right)^T$
- ○ $\left(\frac{25}{12}, \frac{1}{6}, \frac{25}{12}\right)^T$

Exercise 4: For the matrix A given by

$$A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 17/4 & 11/4 \\ 1 & 11/4 & 7/2 \end{pmatrix},$$

show that Cholesky matrix L such that $A = LL^T$, is given by

$$L = \begin{pmatrix} 2 & 0 & 0 \\ -1/2 & 2 & 0 \\ 1/2 & 3/2 & 1 \end{pmatrix}.$$

Exercise 5: Consider the linear system $A\boldsymbol{x} = \boldsymbol{b}$, with

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}.$$

What is the second iterate of the Jacobi scheme, $\boldsymbol{x}^{(k+1)} = \left(I - D^{-1}A\right)\boldsymbol{x}^{(k)} + D^{-1}\boldsymbol{b}$, where D is the diagonal matrix of A and $\boldsymbol{x}^{(0)} = (0, 0, 0)^T$?

- ○ $\left(0, \frac{1}{4}, -\frac{3}{4}\right)^T$
- ○ $\left(\frac{1}{7}, -\frac{5}{14}, \frac{3}{7}\right)^T$
- ○ $(1, 1, 1)^T$
- ○ $\left(-\frac{1}{3}, 0, -\frac{4}{7}\right)^T$
- ○ $\left(\frac{1}{3}, 0, \frac{4}{7}\right)^T$
- ○ $\left(\frac{1}{4}, -\frac{5}{8}, \frac{3}{4}\right)^T$

Exercise 6: Using the Newton formula $x_{n+1} = x_n - f(x)/f'(x)$, show that for $f(x) = \cos(x) - x = 0$, with $x_0 = \pi/4$, that after two iterations the approximation, x_2, to the solution is

- ○ 0.7854
- ○ 0.0167
- ○ 0.8489
- ○ −0.7395
- ○ −0.7489
- ○ 0.7391