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JTMS-MAT-13: Numerical Methods

Sample Multiple Choice Exam Questions

Question 1:
What is the solution after the first step of the Newton method for the function $f(x) = 3x^3 + 5x + 1$ with the initial guess $x_0 = 1$

- ○ $x_1 = 3/4$
- ○ $x_1 = 2/14$
- ○ $x_1 = 1/4$
- ○ $x_1 = 9/10$
- ○ $x_1 = 5/14$

Question 2:
What conditions need to hold for the bisection method to converge to a root of the function $f(x)$ within the interval $[a, b]$?

- ○ $f \in \mathcal{C}^2([a, b])$
- ○ $f' \neq 0$
- ○ $f \in \mathcal{C}^1([a, b])$
- ○ $f(a)f(b) < 0$
- ○ $f \in \mathcal{C}([a, b])$
- ○ $f(a) < f(b)$

Question 3:
What are the correct values for the coefficients a and b for the least squares approximation for $h(x) = a + bx^2$ and the data pairs $(-1, 1)$, $(0, 2)$ and $(1, 1)$?

- ○ $a = 1$
- ○ $a = -1$
- ○ $b = 2$
- ○ $b = -2$
- ○ $b = -1$
- ○ $b = 1$
- ○ $b = 0$
- ○ $a = 0$
- ○ $a = -1$
- ○ $a = 1$
- ○ $a = 2$
- ○ $a = -2$

Question 4:
For the following Runge-Kutta method:

$$u_{n+1} = u_n + (h/6)\left(K_1 + 4K_2 + K_3\right)$$
$$\text{with} \quad K_1 = f(u_n, t_n), \quad K_2 = f(u_n + hK_1/2, t_n + h/2), \quad \text{and} \quad K_3 = f(u_n - hK_1 + 2hK_2, t_n + h)$$

select the correct Butcher array

○ $\begin{array}{c|ccc} 1 & & & \\ 4 & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & -1 & 2 & \end{array}$

○ $\begin{array}{c|ccc} & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & 1 & 4 & 1 \end{array}$

○ $\begin{array}{c|ccc} \frac{1}{6} & & & \\ \frac{2}{3} & \frac{1}{2} & & \\ \frac{1}{6} & -1 & 2 & \\ \hline & -1 & 2 & \end{array}$

○ $\begin{array}{c|ccc} & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & & 2 & \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$

○ $\begin{array}{c|ccc} & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$

Question 5:
For which of the following matrices can you perform a Cholesky decomposition?

○ $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 8 \end{pmatrix}$.

○ $\begin{pmatrix} 2 & 1 & 7 \\ 2 & 2 & 3 \\ 2 & 2 & 1 \end{pmatrix}$

○ $\begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}$.

○ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

Question 6:
Which of the following systems is/are overdetermined?

○ $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 8 \end{pmatrix}$

○ $\begin{pmatrix} 2 & 1 & 7 \\ 2 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix}$

○ $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 5 & 1 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

Question 7:
Given the following data:

i	0	1	2
x_i	0	1	3
y_i	1	3	2

Using polynomial interpolation, what is the value of $y(2)$?

○ $y(2) = 3/2$ ○ $y(2) = 3/4$ ○ $y(2) = 10/3$

○ $y(2) = 11/4$ ○ $y(2) = 4/9$ ○ $y(2) = 0$

Question 8:
Using the trapezoidal rule with three subintervals, approximate the integral

$$T = \int_0^4 x^2 + 1 \,\mathrm{d}x$$

and find the approximate solution as

- ○ $T = 29.000$
- ○ $T = 19.333$
- ○ $T = 30.100$
- ○ $T = 28.519$
- ○ $T = 25.650$
- ○ $T = 26.519$

Question 9:
Given the system of non-linear equations

$$f(x_1, x_2) = \begin{pmatrix} 2x_1 + \cos(x_2) \\ x_1^3 + x_1 \cos(x_2) \end{pmatrix}$$

what is the Jacobian matrix that needs to be inverted for the Newton method?

- ○ $\begin{pmatrix} 2 & -\sin(x_2) \\ 3x_1^2 + \cos(x_2) & -x_1 \sin(x_2) \end{pmatrix}$
- ○ $\begin{pmatrix} -\sin(x_2) & -x_2 \sin(x_2) \\ 3x_1 & 2 \end{pmatrix}$
- ○ $\begin{pmatrix} 2 & -\sin(x_2) \\ 3x_1^2 + \cos(x_2) & x_1 \cos(x_2) \end{pmatrix}$
- ○ $\begin{pmatrix} 2 & 3x_1^2 \\ \cos(x_2) & -x_1 \sin(x_2) \end{pmatrix}$
- ○ $\begin{pmatrix} 2x_1 & 2x_2 \\ \sin(x_1) & \cos(x_2) \end{pmatrix}$
- ○ $\begin{pmatrix} -\sin(x_2) & 2 \\ -x_1 \sin(x_2) & 2x_1 \end{pmatrix}$

Question 10:
Which of the following statements is true?

- ○ The order of convergence of the bisection method is 1.
- ○ The optimal order of convergence of secant method is higher than that for Newton method.
- ○ Bisection method will always find a root if the function is continuous.
- ○ Newton method will always find a root if the derivative exists and is not equal to zero.
- ○ For convergence in $[a, b]$, bisection method needs a continuous function on $[a, b]$, and $f(a)f(b) < 0$.
- ○ Under certain conditions, Newton method has quadratic convergence.

Question 11:
Select those statements that are correct.

- ○ Forward Euler has a higher order than Backward Euler.
- ○ Runge-Kutta schemes are explicit if the a_{ij} coefficients of the Butcher array are zero for all entries along and above the diagonal.
- ○ Backward Euler is implicit and second order accurate.
- ○ Heun's method is second order accurate and explicit.
- ○ The Crank-Nicolson method is implicit.
- ○ The Crank-Nicholson and Backward Euler methods are both second order accurate.

Question 12:
What are the values of approximation u_1 and u_2 using two iterations of the Backward Euler method for the ordinary differential equation $y' = 2y - 2$ with initial condition $y(0) = 1$ and step size $h = 0.1$.

○ $u_2 = 2$ ○ $u_1 = -1$ ○ $u_1 = 1$

○ $u_1 = 1.5$ ○ $u_2 = 1$ ○ $u_2 = 0.5$

Question 13:
Given the following data:

i	0	1	2
x_i	0	2	4
p_i	2	1	2

Using Newton interpolation, which is the right collocation matrix?

○ $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & 4 & 8 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 8 & 0 & 0 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 4 & 8 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{pmatrix}$