

# MECH1010 : Modelling and Analysis in Engineering I: Integration

## Questions for End of Year Test

### Section A

1. By completing the square or otherwise, show that

$$\int \frac{dx}{\sqrt{2x^2 + 6x + 15}} = \sqrt{\frac{2}{21}} \tan^{-1} \left( \frac{2x + 3}{\sqrt{21}} \right) + c \quad [4]$$

2. Evaluate the definite integral

$$I = \int_2^3 \frac{-2x}{a^2 - x^2} dx. \quad [4]$$

### Section B

3. Consider the ellipse given by the parametric formula  $x = a \sin t$  and  $y = b \cos t$ , then

(i) Show that area of an ellipse is given by  $\pi ab$ . [4]

(ii) Hence or otherwise show that the volume of revolution when the ellipse is fully rotated about the  $x$ -axis is  $4\pi ab^2/3$ . [4]

(iii) Show that the first moment of area about the  $yz$ - and  $xz$ -planes for the quarter ellipse, in the positive quadrant bounded by  $x$  and  $y$ -axes, are given by

$$I_{yz} = a^2 b \int_0^{\pi/2} \sin t \cos^2 t dt \quad \text{and} \quad I_{xz} = -ab^2 \int_{\pi/2}^0 \sin^2 t \cos t dt. \quad [6]$$

- (iv) Hence show that

$$I_{yz} = \frac{a^2 b}{3} \quad \text{and} \quad I_{xz} = \frac{ab^2}{3}. \quad [2]$$