

# MECH1010 : Modelling and Analysis in Engineering I: Integration

## Questions for Mid Year Test

### Section A

1. Let  $f(x) = \sin x$ , then find  $f'(x)$  and derive the known results

(i)  $\int f(x) f'(x) dx,$   
(ii)  $\int f'(x) / f(x) dx.$

[2]  
[2]

2. Using partial fractions find

$$I = \int \frac{dx}{x^2 - a^2}. \quad [4]$$

3. A cycloid is the curve defined by the path of a point on the edge of circular wheel as the wheel rolls along a straight line. It is given in parametric form by  $x = r(t - \sin t)$  and  $y = r(1 - \cos t)$ . Show that the arc-length given by one complete rotation is given by

$$I = 2r \int_0^{2\pi} \sin \frac{t}{2} dt = 8r. \quad [4]$$

### Section B

4. (i) Derive the following reduction formula

$$I_n = \int x^n e^{-x} dx = -x^n e^{-x} + nI_{n-1}. \quad [4]$$

- (ii) Hence or otherwise show that the first moment of area about the  $yz$ -plane for the curve defined by  $f(x) = e^{-x}$  between the  $x$ - and  $y$ -axes and the line  $y = 1$  is  $1 - 2e^{-1}$ . [4]

- (iii) Hence or otherwise find the second moment of area about the  $y$ -axis for the curve defined by  $f(x) = e^{-x}$  between the  $x$ - and  $y$ -axes and the line  $y = 1$ . [4]

- (iv) Using Simpson's rule with five equally spaced element to find an approximation to first moment of area about the  $yz$ -axis. [8]