MECH1010 : Modelling and Analysis in Engineering I: Integration

Problem Sheet 1

Section A

1. Evaluate the following definite integrals

   (i) $\int_{-3}^{3} x^2 \, \mathrm{d}x$

   (ii) $\int_{-1}^{1} x \left(x^2 + 1\right) \, \mathrm{d}x$.

2. Sketch the curve $y = 2x \left(x - 1\right) \left(x + 1\right)$, indicating all roots and turning points. What is the integral over the range $x = -1$ to $x = 1$? What is the value of area defined by this curve?

3. Find the following indefinite integrals

   (i) $\int \sin \left(5x + 3\right) \, \mathrm{d}x$

   (ii) $\int \left(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1\right) \, \mathrm{d}x$

   (iii) $\int \left(x + 1\right)^7 \, \mathrm{d}x$

   (iv) $\int \dfrac{\mathrm{d}x}{\sqrt{16 + 4x^2}}$

   (v) $\int \dfrac{\mathrm{d}x}{\sqrt{x^2 + 2x + 17}}$.

Section B

4. Find the following indefinite integral

$$\int \left(2x^2 - \frac{1}{x}\right)^2 \, \mathrm{d}x.$$

5. For the function $f \left(x\right) = x \left(x + a\right) \left(x - b\right)$ for what relative values of $a > 0$ and $b > 0$ is the integral $\int_{-a}^{b} f \left(x\right) \, \mathrm{d}x$ positive and when is it negative?

6. Find the area between the two curves $y_1 = x^2 + 2$ and $y_2 = 4 - x^2$.

7. A torus can be defined as a circle of radius r centred at a distance $R > r$ from the x- and y-axes, rotated about 2π radians. What is the volume of a torus?