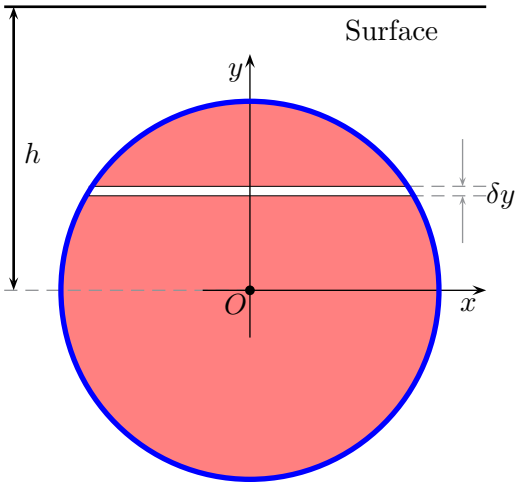
MECH1010 : Modelling and Analysis in Engineering I: Integration

Problem Sheet 2*

Section A

1. A solid of constant density, ρ, is formed by a rotation about the x-axis of the area bounded by the x-axis, the lines $x = 1$, $x = 3$ and the curve $y = 2x - 1$. What are the coordinates of the centre of gravity?

2. A circular plate of radius a is submerged at a depth h in a fluid.



The second moment of inertia can be found by taking second moment of area about the plate's centreline along the x-axis and applying the parallel axis theorem.

(i) By defining a suitable area element as $\delta A = 2x\delta y$, show that the second moment of area about the centreline can be expressed as $I_{AO} = 4\int_0^R y^2 x \, \mathrm{d}y$.

(ii) Changing into polar coordinates $x = R\cos\theta$, $y = R\sin\theta$, show that second moment of inertia about the centreline can be expressed as

$$I_{AO} = 4\int_0^{\pi/2} \sin^2\theta\cos^2\theta \, \mathrm{d}\theta.$$

(iii) On evaluating the integral, apply the parallel axis theorem to show that $I_h = \pi R^2 h^2 + \pi R^4/4$.

Section B

3. A torus can be defined as a circle of radius r centred at a distance $R > r$ from the x- and y-axes, rotated about 2π radians. What is the surface area of a torus?

4. A catenary is the theoretical shape a hanging chain or cable will assume when supported at its ends and acted on only by its own weight. A catenary takes the form $y = b + a\cosh(x/a)$. Thus if a cable whose shape takes the form of a catenary with $a = 1$ and $b = 6.22$ is suspended between $x = -1$ and $x = 1$ what is the length of the cable?

*This document can be downloaded from: http://www.ucl.ac.uk/~ucesdsi/teaching.html