MECH1010 : Modelling and Analysis in Engineering I: Integration

Problem Sheet 3*

Section A

1. Evaluate
   (i) $\int_0^1 x^3 e^x \, \mathrm{d}x$
   (ii) $\int_0^{\pi/2} \cos^4 \theta \, \mathrm{d}\theta$.
2. Find $\int \dfrac{x}{1+x} \, \mathrm{d}x$ using partial fractions and solve the resulting integrals.
3. For $f(x) = xe^x$ find the integrals
   (i) $\int f(x) f'(x) \, \mathrm{d}x$
   (ii) $\int f(x) / f'(x) \, \mathrm{d}x$.

Section B

4. Evaluate the following definite integral
$$\int_{-1}^{1} \frac{\ln(3+2x)}{3+2x} \, \mathrm{d}x.$$
5. Find the following indefinite integrals
   (i)
$$\int \frac{3x^2-1}{x(x^2-1)} \mathrm{d}x,$$
   (ii)
$$\int \frac{\mathrm{d}x}{3\sin^2 x - 5\cos^2 x}.$$
6. Show that
$$\int \sin^n x \cos^m x \, \mathrm{d}x = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, \mathrm{d}x$$
   and
$$\int \sin^n x \cos^m x \, \mathrm{d}x = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \, \mathrm{d}x.$$
7. Using either Simpson's or the Trapezium rule, by dividing the range into ten equally sized parts evaluate $\int_0^1 e^{-x^2} \, \mathrm{d}x$.

*This document can be downloaded from: http://www.ucl.ac.uk/~ucesdsi/teaching.html