

MECH1010 : Modelling and Analysis in Engineering I: Linear Algebra

Problem Sheet 1*

For the following questions let

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{d} = \begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix}.$$

1. Find the vectors representing the following:

- (i) $\hat{\mathbf{a}}$
- (ii) $3\mathbf{b} - \mathbf{c}$

and the scalars for

- (i) $\mathbf{a} \cdot \mathbf{b}$
- (ii) The angle between \mathbf{a} and \mathbf{b} .

2. Find the point of intersection of the line through the origin and the point $(1, 2, 1)$ with the plane containing the points $(1, 0, -1)$, $(0, -2, 0)$ and $(0, 0, 4)$.

3. Find the work done when a force of 6N acting in the direction \mathbf{f} moves an object from $(1, 3, 5)$ to the origin. Where $\mathbf{f} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and the units are in metres.

4. Find the minimum distance from the point P to the plane Π where P has the position vector $\mathbf{p} = (4, 2, 5)$ and Π contains the origin and has a normal vector $\mathbf{n} = (2, -2, 4)^T$.

5. Find

- (i) $\mathbf{a} \times \mathbf{c}$
- (ii) $\mathbf{b} \times \mathbf{c}$
- (iii) The area of the triangle which has sides \mathbf{a} and \mathbf{c}
- (iv) The equation of the plane containing the (direction) vectors \mathbf{a} and \mathbf{b} and passing through the point P which has position vector \mathbf{d} . Express the answer in both Cartesian and vector form.

6. Given two planes Π_1 and Π_2 , given by

$$\Pi_1 : x - 2y - 2z = 10$$

$$\Pi_2 : y = 2(3z - x)$$

find the acute angle between them and the equation of the line of intersection.

*This document can be downloaded from: <http://www.ucl.ac.uk/~ucesdsi/teaching.html>