MECH1010 : Modelling and Analysis in Engineering I: Linear Algebra

Problem Sheet 2*

1. A ray has equation
$$\boldsymbol{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}.$$
Where, if at all, will it strike the sphere with radius 3 and centre $(3, -1, 3)$?

2. A tetrahedron has vertex with co-ordinates; [Q.14 1993]
$$A = (0, 1, 2), \quad B = (1, 3, 5), \quad C = (2, -1, 3) \quad \text{and} \quad D = (1, 3, 1)$$
   (i) Find the angle between lines $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (ii) Find the area of face ABC.
   (iii) What is the distance of D from the plane containing the face ABC as measured parallel to the z-axis.
   (iv) Calculate the volume of the tetrahedron.

3. Find $\boldsymbol{x}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ given that
$$\boldsymbol{a} \cdot \boldsymbol{b} \neq 0, \quad \boldsymbol{b} \cdot \boldsymbol{x} = 1 \quad \text{and} \quad |\boldsymbol{a} \times \boldsymbol{x}| = 0.$$

4. Find the minimum distance between the following pairs of lines;
   (i) $x - 1 = -\dfrac{y+6}{4} = \dfrac{3-z}{5}$ and $\dfrac{1-2x}{2} = \dfrac{y-1}{4} = \dfrac{z}{5}$.
   (ii) $x = y + 2 = 6z - 6$ and $\boldsymbol{r} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 6 \\ -1 \end{pmatrix}$.

5. For both of the pairs of lines given in question 4, find the equation of the line (or family of lines) that intersects the pair of lines and is mutually perpendicular to both of the lines in each pair..

6. Let
$$\boldsymbol{A} = \begin{pmatrix} 1 & 3 & -4 \\ 0 & 1 & -1 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 0 & 4 \\ -2 & 1 \\ 3 & -1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{C} = \begin{pmatrix} 1 & 4 \\ 6 & 3 \end{pmatrix}.$$
How many possible ways of multiply any two of the matrices together? Find all possible combinations. How many ways are there if the transposes of the matrices are considered?

7. Let
$$\boldsymbol{i} = (1, 0), \quad \boldsymbol{j} = (0, 1) \quad \text{and} \quad \boldsymbol{R}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$
   (i) Show the vectors are orthogonal.
   (ii) Find $\boldsymbol{v}_1 = \boldsymbol{R}(\theta)\,\boldsymbol{i}$.
   (iii) Show that $\hat{\boldsymbol{v}_1} = \boldsymbol{v}_1$
   (iv) Find $\boldsymbol{v}_1 \cdot \boldsymbol{j}$.
   (v) Find $\boldsymbol{v}_2 = \boldsymbol{R}(\theta)\,\boldsymbol{j}$ and show that $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are orthogonal.

*This document can be downloaded from: http://www.ucl.ac.uk/~ucesdsi/teaching.html