MECH1010 : Modelling and Analysis in Engineering I: Vectors

Test : Thursday 24$^{\text{th}}$ February 2011

Time allowed : **50** minutes.

This is an open book test; you may use your lecture notes, exercise sheets and any reference books but no electronic aides such as laptops. Scientific caluculators are permitted.

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1. For $\boldsymbol{a} = (1, 3, 5)$, $\boldsymbol{b} = (-1, 4, -5)$ and $\boldsymbol{c} = (1, 0, 2)$ find $(\boldsymbol{b} \cdot \boldsymbol{a})\boldsymbol{c} + (\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b}$. [2]

2. The three points $(4, 3, 2)$, $(5, 0, 5)$ and $(1, 1, 1)$ form the vertices of a triangle.
   (i) Find the cosine of the angle between the sides of the triangle which meet at $(1, 1, 1)$. [3]
   (ii) Find the area of the triangle. [3]

3. Two unit vectors $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ have an angle θ, $(0 \leq \theta \leq \pi/2)$, between them. Let

$$\boldsymbol{c} = \hat{\boldsymbol{b}} - \left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}}\right) \hat{\boldsymbol{a}}.$$

   (i) Show that $\boldsymbol{c}$ is orthogonal to $\hat{\boldsymbol{a}}$. [2]
   (ii) By calculating $\boldsymbol{c} \cdot \boldsymbol{c}$, or otherwise, show that the length of $\boldsymbol{c}$ is given by $\sin\theta$. [2]

4. (i) Show that the equation of the line through the points $(1, 2, 1)$ and $(3, 0, 4)$ can be written as

$$\frac{x-3}{-2} = \frac{y}{2} = \frac{z-4}{-3} = \lambda. \quad [2]$$

   (ii) Show that the equation of the plane containing the points $(1, 1, 1)$, $(0, -2, 0)$ and $(1, 0, 4)$ can be written as

$$10x - 3y - z = 6. \quad [3]$$

   (iii) Find the point of intersection between the plane and the line. [2]
   (iv) Is the point of intersection nearer $(1, 2, 1)$ or $(3, 0, 4)$? [2]

5. Using the scalar triple product, or otherwise, find the volume of the parallelepided form by the three vectors $\boldsymbol{a} = (2, 0, 0)$, $\boldsymbol{b} = (3, 1, 0)$ and $\boldsymbol{c} = (1, 1, 5)$. [4]

6. For the pair of planes Π_1 and Π_2, given by

$$2x + y - 3z = 8 \quad \text{and} \quad \alpha x + y + z = 3.$$

   (i) Determine the value of α so that the planes are orthogonal. [2]
   (ii) Show that the line of intersection between the two planes can be written as

$$\boldsymbol{r} = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix}. \quad [3]$$